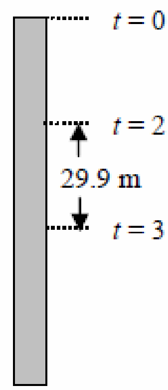
2007 Linear Motion Question

1. **(a)** A particle is projected vertically downwards from the top of a tower with speed u m/s. It takes the particle 4 seconds to reach the bottom of the tower.



During the third second of its motion the particle travels 29.9 metres.

Find

(i) the value of u

(ii) the height of the tower.

(b) A train accelerates uniformly from rest to a speed v m/s.

It continues at this speed for a period of time and then decelerates uniformly to rest.

In travelling a total distance d metres the train accelerates through a distance pd metres and decelerates through a distance qd metres, where $p < 1$ and $q < 1$.

(i) Draw a speed-time graph for the motion of the train.

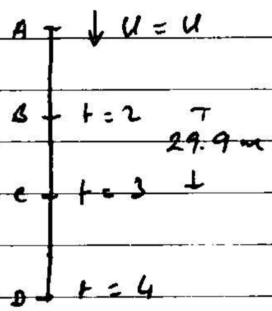
(ii) If the average speed of the train for the whole journey is $\dfrac{v}{p+q+b}$,

find the value of b.

2007

Q.1

(a) (i)



$\underline{A \rightarrow B}$

$u = u$

$v = -$

$a = g$

$s = x$

$T = 2$

$$S = uT + \tfrac{1}{2}aT^2$$
$$x = u(2) + \tfrac{1}{2}(g)(2)^2$$
$$\boxed{x = 2u + 2g}$$

5

$\underline{A \rightarrow C}$

$u = u$

$v = -$

$a = g$

$s = x + 29.9$

$T = 3$

$$S = uT + \tfrac{1}{2}aT^2$$
$$x + 29.9 = u(3) + \tfrac{1}{2}(g)(3)^2$$
$$x + 29.9 = 3u + \tfrac{1}{2}(g)(9)$$
$$\boxed{x + 29.9 = 3u + 4.5g}$$

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$$2u + 2g + 29.9 = 3u + 4.5g$$
$$2g + 29.9 - 4.5g = 3u - 2u$$
$$29.9 - 2.5g = u$$
$$29.9 - 24.5 = u$$
$$\underline{\underline{5.4 \text{ m/s} = u}}$$

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(ii) $\underline{A \rightarrow D}$

$u = 5.4$

$v = -$

$a = g$

$s = ?$

$T = 4$

$$S = uT + \tfrac{1}{2}aT^2$$
$$S = (5.4)(4) + \tfrac{1}{2}(g)(4)^2$$
$$S = 21.6 + 8g$$
$$S = 21.6 + 78.4$$
$$\underline{\underline{S = 100 \text{ m}}}$$

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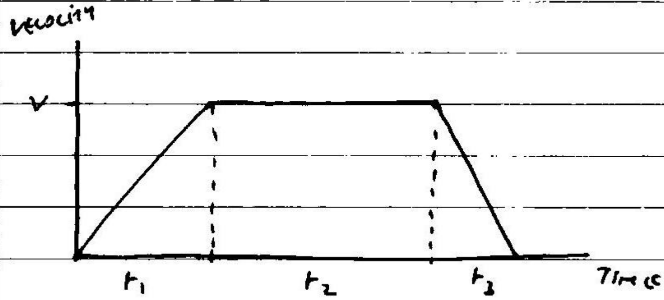
(b)

(i)



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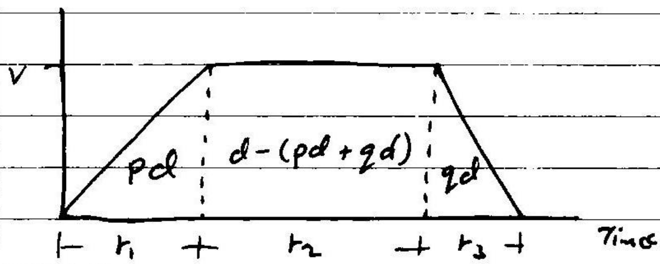
(ii)



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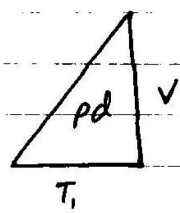

$$pd = \frac{1}{2}(T_1)(V)$$

$$\boxed{\frac{2pd}{V} = T_1}$$

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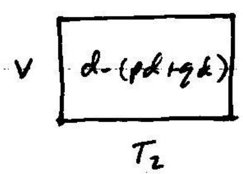
②



$$d-(pd+qd) = (T_2)(V)$$

$$d-pd-qd = T_2 V$$

$$\boxed{\frac{d(1-p-q)}{V} = T_2}$$

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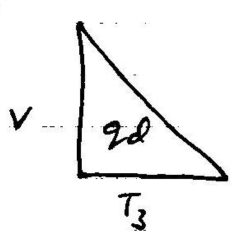
③



$$qd = \frac{1}{2}(T_3)(V)$$

$$\boxed{\frac{2qd}{V} = T_3}$$

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$$\text{AVERAGE SPEED} = \frac{\text{TOTAL DISTANCE}}{\text{TOTAL TIME}}$$

$$\text{AVERAGE SPEED} = \frac{d}{\left(\frac{2dp}{v}\right) + \left(\frac{d - dp - dq}{v}\right) + \left(\frac{2dq}{v}\right)}$$ (5)

$$\text{AVERAGE SPEED} = \frac{d}{\frac{2dp + d - dp - dq + 2dq}{v}}$$

$$\text{AVERAGE SPEED} = \frac{d}{\frac{dp + dq + d}{v}}$$

$$\text{AVERAGE SPEED} = \frac{d}{\frac{d(p+q+1)}{v}}$$

$$\text{AVERAGE SPEED} = \frac{d}{d(p+q+1)} \cdot \frac{v}{1} = \frac{v}{p+q+1}$$

$$\frac{v}{p+q+b} = \frac{v}{p+q+1}$$ (5)

$$\therefore \underline{\underline{b = 1}}$$